Design for Transonic Flight – The Area Rule

Chapter 8

The performance of rockets designed to fly through the transonic range may be improved by using the Area Rule. A design with Area Rule may squeak out a few more feet of altitude or a few more miles per hour in maximum speed. Design with Area Rule might be used when trying to break altitude records. So, what is Area Rule? Many jet aircraft and missiles exhibit a "coke bottle" shape or waist in the fuselage (body) at the wing-to-fuselage junction or fin-to-body junction. This waist is a good clue that the aircraft or missile is designed to operate in the transonic region, typically $0.80 \leq$ Mach Number ≤ 1.2. The waist is a result of applying the concept of Area Rule to minimize wave drag-rise at a given operational design Mach Number within the transonic region. Figure 1 illustrates the difference in wave drag-rise over the transonic region for a wing-body combination with and without Area Ruling.

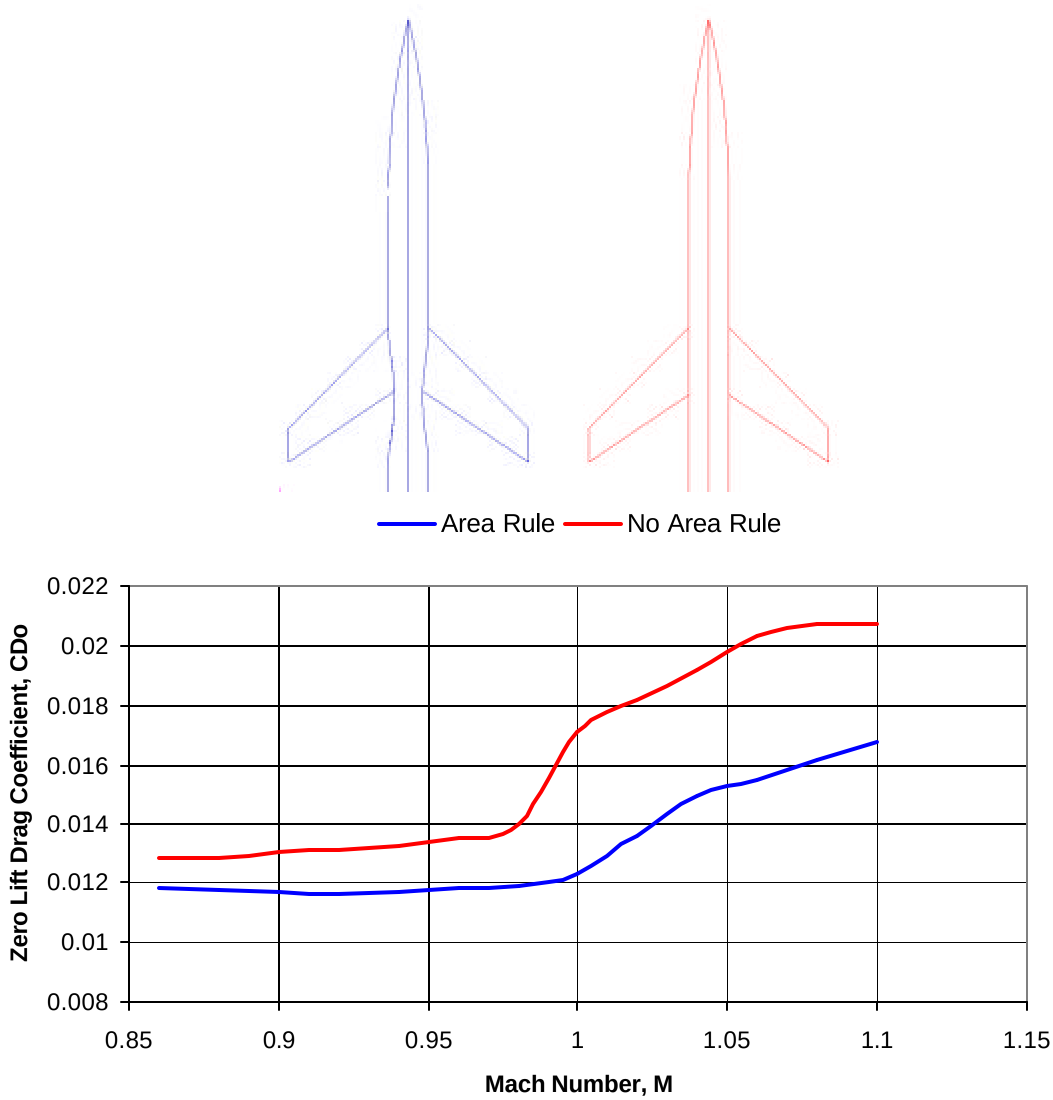


Figure 1 – Zero-Lift Drag, comparing a configuration with Area Rule to one without Area Rule (See ref 1, Whitcomb)

It is apparent from Figure 1 that the "coke bottle" shape body produces significantly less zero-lift drag over the transonic region. This is largely due to a reduction in transonic wave drag. Also, it is apparent that the Divergent Mach Number (M_D) is shifted to a slightly higher value. Recall from Chapter 1 that the Divergent Mach Number is that Mach Number characterized by a sudden increase in drag due to wave-drag associated with compression or shock waves. A method to estimate the reduction in wave-drag rise and this shift in Divergent Mach Number will be discussed later in this chapter.

Supersonic Slender Body Theory suggests that a body of revolution with an equivalent longitudinal distribution of cross-sectional area can reproduce far-field pressure disturbances caused by a wing-body combination. Whitcomb (ref 1) found this to be true for bodies and wing-body combinations at zero angle-of-attack operating in the transonic region. Two different wing-body and/or body combinations that produce similar far-field pressure disturbances will produce similar drag. Thus, for a given body of revolution that produces minimum drag, then there might be a practical wing-body combination that produces an equivalent minimum drag. According to Slender Body Theory a body must have a smooth continuous axial (longitudinal) distribution of cross-sectional area for minimum drag; that is, the derivative of the longitudinal area distribution must be a smooth continuous function. Experimental observations by Whitcomb were in agreement with this theory. The classical Sears-Haack body will produce a minimum wave-drag rise. The Sears-Haack body can be approximated by a full ogive, and this will be the approach adopted here.

The Area-Rule is simply the transformation of a minimum drag body of revolution (i.e.; Sears-Haack or ogive) to a practical wing-body combination, hence the resulting "coke-bottle" shape. The Area-Rule transformation can be quite complex when designing to supersonic Mach Number flight at nonzero angular attitudes (pitch, roll, and yaw). However, for purposes of applying Area-Rule to rockets in transonic flight, it will be assumed that the rocket is aligned with the direction of flight and that the free-stream design Mach Number will be 1. This approach should provide for a reasonable wave-drag reduction over the transonic range.

Rather than getting bogged down in the sophisticated and complex mathematics of theories such as the Method of Characteristics, Slender Body Theory and Supersonic Area Rule, we will apply the "spirit" of the Area-Rule method to rocket design.

Example 1 – Ogive Empennage

In this example we will begin with the basic full ogive (football shape), as in Figure 2a. We can define the length of the ogive as twice the length of the rocket's nose cone, and its maximum diameter as the base diameter of the nose cone. Most rockets require a payload section, a recovery section and an engine section in addition to the nose cone. Usually the ogive shape will not have sufficient volume to accommodate these sections. Thus, we can add a cylindrical body tube between the forward and aft cones of the ogive, as in Figure 2b. Finally, we will have to truncate the aft cone to allow an exit for engine exhaust. The result is shown in Figure 2c.

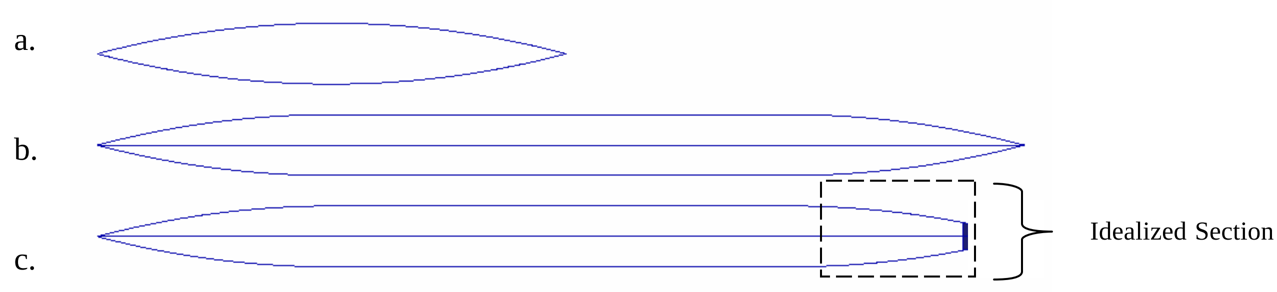


Figure 2 – The evolution of the rocket body from a simple ogive, before applying Area-Rule.

The idealized section, as enclosed by the dashed line of Figure 2c, is the shape we wish to approximate with an equivalent fin and body combination. This truncated half-ogive can be described by a length 'l_e', a base radius 'r_o', an exit radius 'r_e', and a characteristic radius 'R'. The characteristic radius 'R' can be calculated from the other dimensions that are specified by the rocket designer. Figure 3 describes this geometry.

Eqn 8.1

$$R = \frac{\left(r_O^2 - r_e^2\right) + l_e^2}{2\left(r_O - r_e\right)} = \text{Characteristic Radius}$$

l_e = length of truncated half-ogive
r_O = base radius of truncated half-ogive, typically ½-body tube diameter
r_e = exit radius of truncated half-ogive

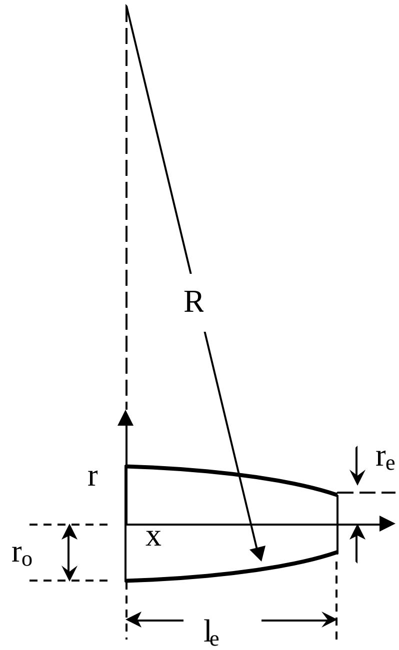


Figure 3 – Geometry of the idealized section (truncated half-ogive)

A local coordinate system is also depicted in Figure 3 above with vertical axis 'r' and horizontal axis 'x'. 'x' is the longitudinal coordinate originating from the base of the half-ogive, and 'r' is the local radius of the section at given 'x'. Note that $x = 0$ at the base of the truncated half-ogive and $x = l_e$ at the exhaust end of the truncated half-ogive. Also, $r = r_o$ at $x = 0$ and $r = r_e$ at $x = l_e$. The local radius is a function of the longitudinal coordinate 'x', and is given as follows:

Eqn 8.2

$$r(x) = r_O - R\left\{1 - \cos\left[\sin^{-1}\left(\frac{x}{R}\right)\right]\right\}$$, Local radius of optimized section

Now that we have defined a relation for the idealized shape, let us define a relation for an equivalent wing and body combination that yields the same longitudinal distribution of cross-sectional area. In reference to Figure 4 below, we are in search of a relation defining the body geometry given by $e(x)$, which in combination with the fins, yields the same cross-section area distribution as that of the idealized shape given by the dash line of Figure 4.

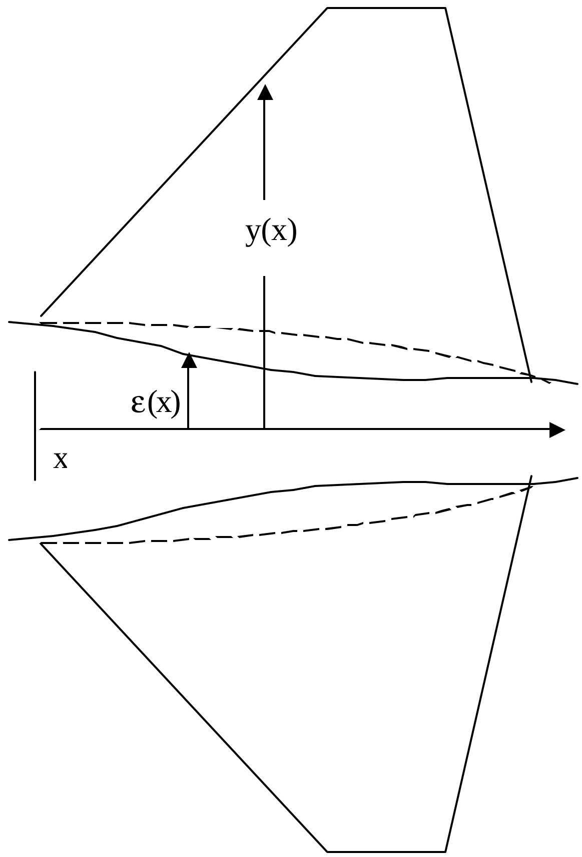


Figure 4 – Geometry of wing and body combination that yields equivalent longitudinal area distribution as the idealized section (dash line).

The equation describing $e(x)$ will not be derived here, but will be presented along with a short discussion of the pertinent variables. First, the number of fins must be known (typically 3 or 4). Second, the average thickness of the exposed fin at each longitudinal location 'x' must be known. The exposed fin is the fin surface outside the boundaries of the rocket's body. In many cases rocket fins are constructed of constant thickness stock. Finally, the local fin half-span taken from the body longitudinal axis must be known. The equation for $e(x)$ is given below in terms of the quantities just described.

Eqn 8.3

$$e(x) = \frac{N_F t + \sqrt{N_F^2 t^2 - 4p\left(N_F t y - p r^2\right)}}{2p}$$

Where,

N_F = number of fins
t = average thickness of exposed fin, may vary with 'x'
y = fin local half-span taken from the body longitudinal axis, see Figure 4.
r = local radius of idealized section, see Figure 3 and equation 8.2

The above equations were applied to an ogive empennage design called Icarus. This design was expected to have a top speed of Mach 1.25, but never flew with an engine of sufficient impulse to go transonic. The resulting configuration is given as Figure 5.

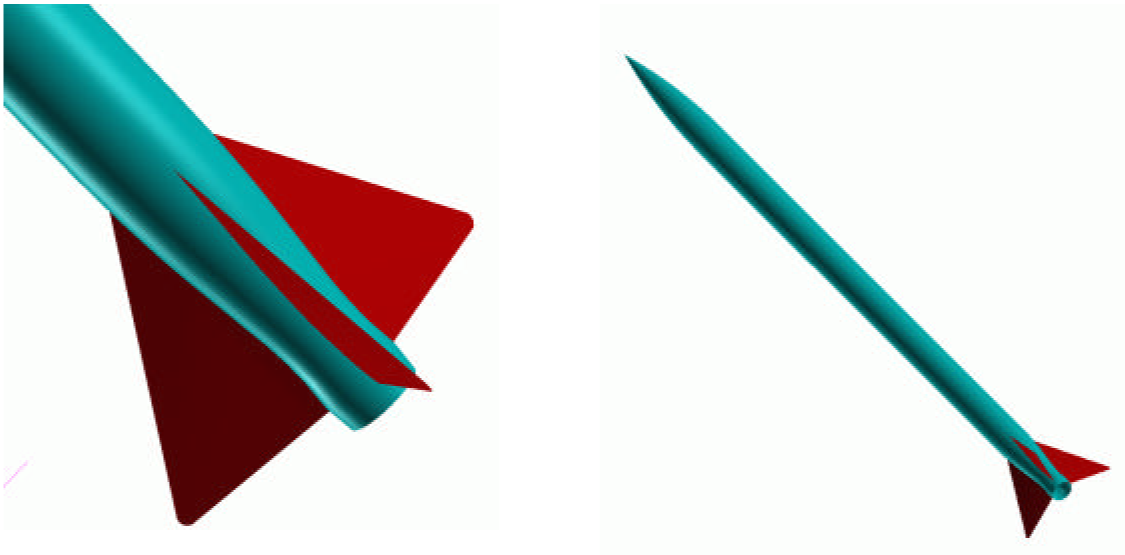


Figure 5 – Icarus, an ogive empennage design with Area Rule

Example 2 – Cylindrical Body

Often the fins will be positioned such that the shape of a half-ogive cannot be approximated. Approximating a constant diameter body may be the best that can be achieved. Figure 6 depicts a delta-fin and body combination with a "coke bottle" or "waist". The dash line of Figure 6 represents the constant diameter body that will be approximated by the fin and "coke bottle" body combination. The "coke bottle" body shape is defined by Equation 8.3, as before, but with the local radius '$r(x)$' replaced by the constant body radius of 'r_o'.

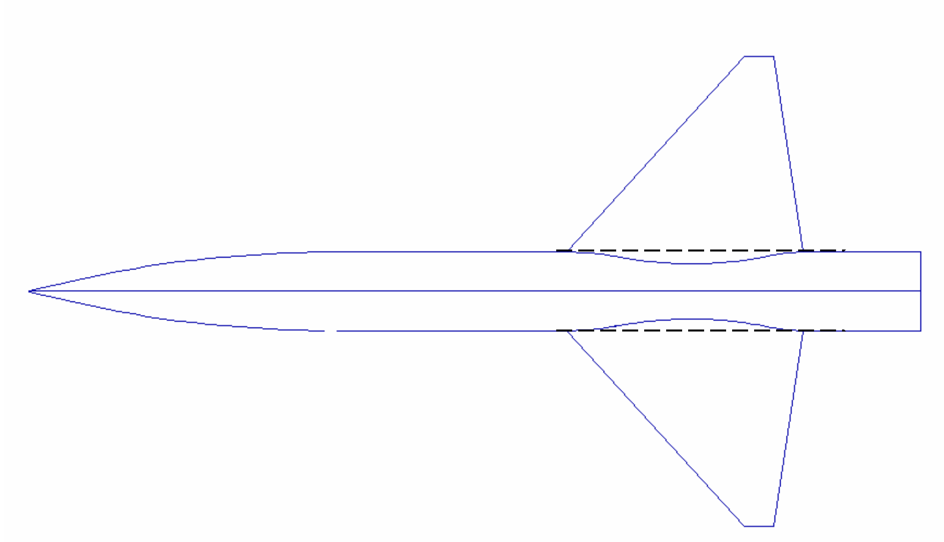
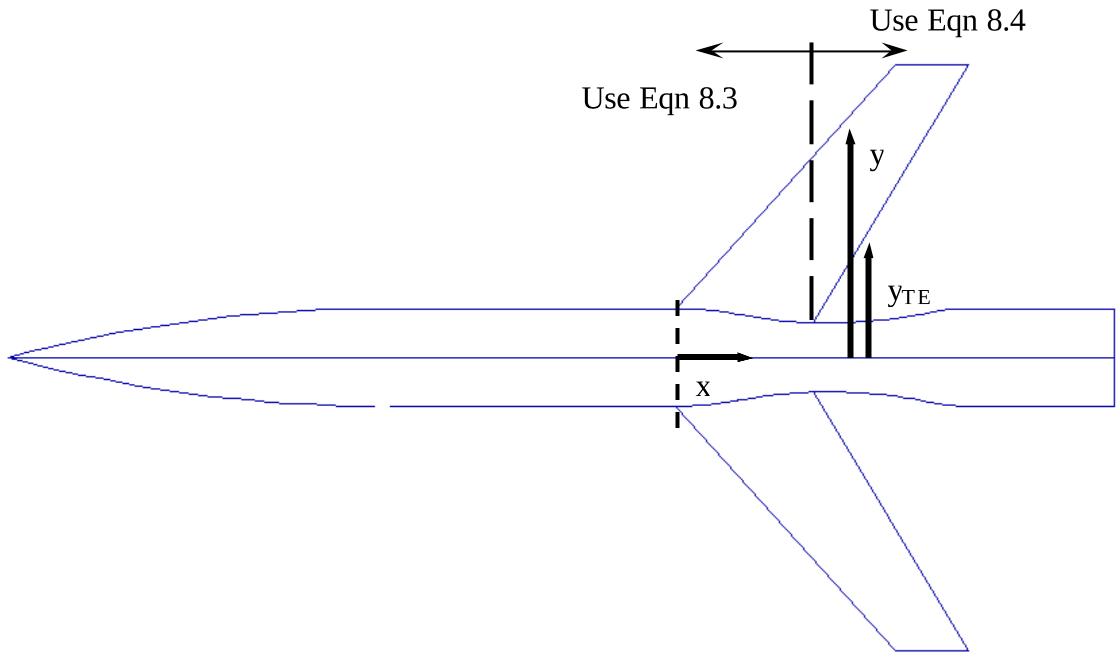


Figure 6 – Geometry of delta-wing and body combination that yields equivalent area distribution to a cylindrical body (dash line).

Example 3 – Highly Swept Wings

For highly swept wings, the trailing edge may sweep well aft of the root chord as illustrated in Figure 7. Equation 8.3 applies only to the portion of the wing-body combination forward of the trailing edge of the root chord. Equation 8.4 below applies to the portion of the wing-body combination aft of this point.



Eqn 8.4

$$e(x) = \sqrt{r^2 - \left(\frac{N_F t}{p}\right)(y - y_{TE})}$$

Where,

r = local radius of equivalent section (example: truncated half-ogive or cylindrical body)
N_F = number of fins
t = average thickness of exposed fin, may vary with 'x'
y = fin local half-span taken from the body longitudinal axis
y_{te} = fin trailing-edge local half span taken from the body longitudinal axis

The equations used in predicting wave drag associated with the methods of Slender Body Theory and Supersonic Area Rule converge to the same result for a Mach Number of 1. We will be consistent with the convention of Chapter 1 by expressing the drag coefficient in terms of the body maximum cross-section area. Thus, the equations to estimate the wave drag rise at M=1 are given below.

$$C_{DW} = \left(\frac{lp}{d}\right)^2 \sum_{n=1}^{\infty} nA_n^2 = \text{Wave Drag Coefficient at Mach 1.0}$$

Where,

$$A_n = \frac{2}{p}\int_0^p \frac{dG}{dx}\sin(nq)dq$$

$$q = \cos^{-1}\left(1 - \frac{2x}{l}\right)$$

$$\frac{dG}{dx} = \frac{1}{lp}\frac{dS}{dx}$$

$$S(x) = pr^2(x) = \text{Cross Section Area Distribution of Equivalent Body}$$

$$l = \text{Length of Equivalent Body}$$

$$r(x) = \text{Body Radius Distribution}$$

$$x = \text{Distance Along Body Longitudinal Axis}$$

$$d = \text{Maximum Diameter of Equivalent Body}$$

The above equations can be quite tricky to use, particularly if the equivalent body area distribution is not a smooth continuous function. As a result, several series terms "A_n" may be required. Even with a sufficient number of terms, the method does a poor job of predicting the wave drag at Mach 1. However, the Supersonic Area Rule has proven to provide reasonable predictions for wave drag at higher supersonic speeds for slender bodies (length-to-diameter ratios on the order of 9 or above), and for slender wing-body combinations with wings of thickness-to-chord ratios of 6% or below.

As an alternative to the above approach, we will adopt a similar approach to that of Chapter 1 for the prediction of wave drag rise over the transonic region. In fact, we will use Equation 1.8 of Chapter 1 exactly. However, the inputs of maximum drag rise (ΔCd_T), transonic drag divergence Mach Number (M_D) and the final transonic Mach Number (M_F) will be defined here specifically for smooth equivalent bodies. Recall that the equivalent body is the idealized body that is approximated by the wing-body combination obtained through Area Ruling. Equation 1.8 of Chapter 1 is restated for your reference.

$\Delta Cd_T =$ *Transonic drag rise for given Mach Number 'M'.*

$$= \Delta C_{DMAX} F, \quad \mathit{If} \; M_D \le M \le M_F$$

$$= 0., \mathit{If} \; M < M_D \; \mathit{or} \; M > M_F$$

Where,

$$F = -8.3474x^5 + 24.543x^4 - 24.946x^3 + 8.6321x^2 + 1.1195x$$

$$x = \left[\frac{(M - M_D)}{(M_F - M_D)}\right]$$

A collection of slender body data, as described in reference 2, was used to develop the following equation to estimate the maximum transonic drag rise ΔC_{DMAX}. Figure 7 is a plot of predicted versus actual wave drag for several slender bodies using this equation. The correlation appears to be reasonable.

$$\Delta C_{DMAX} = me^{n\left(\frac{l}{d}\right)}$$

Where,

$$m = -2.4733\left(\frac{\ln}{l}\right)^3 + 8.7721\left(\frac{\ln}{l}\right)^2 - 7.5995\left(\frac{\ln}{l}\right) + 2.4508$$

$$n = 0.2404\left(\frac{\ln}{l}\right)^2 - 0.2027\left(\frac{\ln}{l}\right) - 0.13$$

$l =$ Length of equivalent body

$\ln =$ Length of nose section of equivalent body

$d =$ Maximum diameter of equivalent body

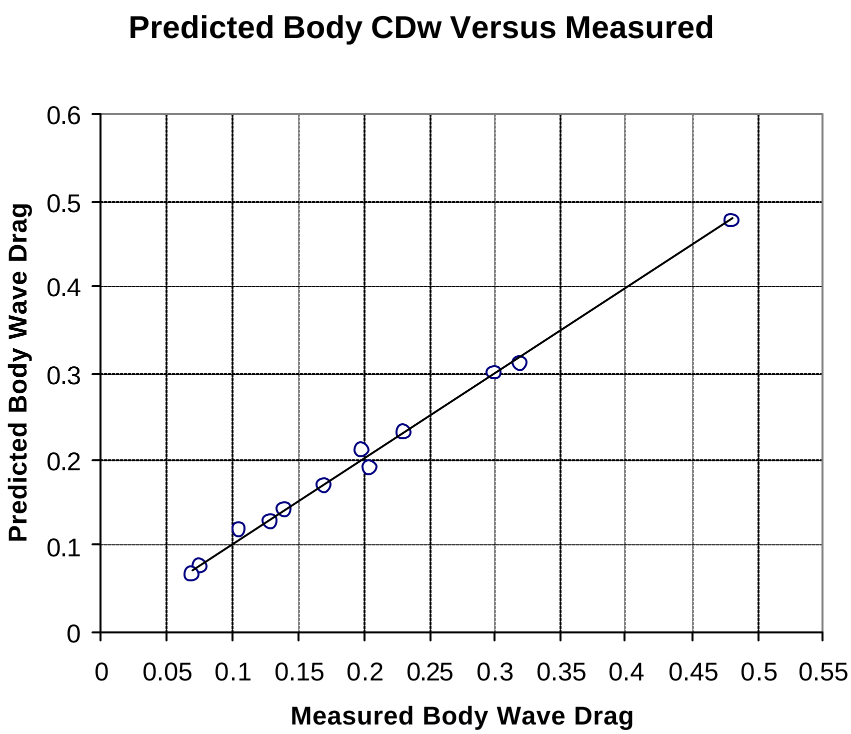


Figure 7 – Correlation of Predicted to Measured Body Wave Drag

The drag divergence Mach Number (M_D) is also estimated by the same parameters. M_D is the Mach Number that is associated with the rise in drag due to shock wave effects. Its value is slightly greater than 0.95 as the body slender ratio (l/d) increases. The equation for the drag divergence Mach Number given below is based on measured data for a limited number of slender bodies of reference 2. The use of this equation should be limited to nose-to-body length ratios of 0.6 or less. Figure 8 shows the agreement of this equation with measured data.

$$M_D = 0.956 - \left(\frac{l}{md}\right)^n, \text{ For } \frac{\ln}{l} \leq 0.6$$

Where,

$$m = 20.928\left(\frac{\ln}{l}\right)^2 - 14.479\left(\frac{\ln}{l}\right) + 4.5882$$

$$n = -22.276\left(\frac{\ln}{l}\right)^2 + 7.5579\left(\frac{\ln}{l}\right) - 2.6693$$

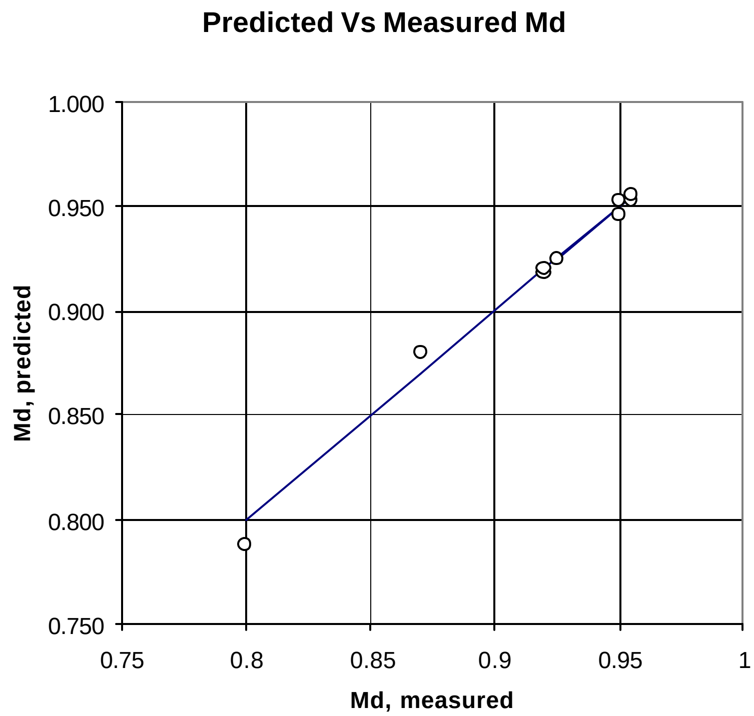


Figure 8 – Correlation of Predicted Drag Divergence Mach Number to Measured.

The last remaining parameter of the drag rise equation needing definition is the Mach Number defining the upper limit of the transonic region. We will denote this Mach Number as the final transonic Mach Number M_F. Again, using the data of reference 2 an equation to predict M_F can be developed as a function of nose-to-body length ratio.

$$M_F = 1.0799 + 0.00117\left(\frac{\ln}{l}\right)^{-3.874}$$

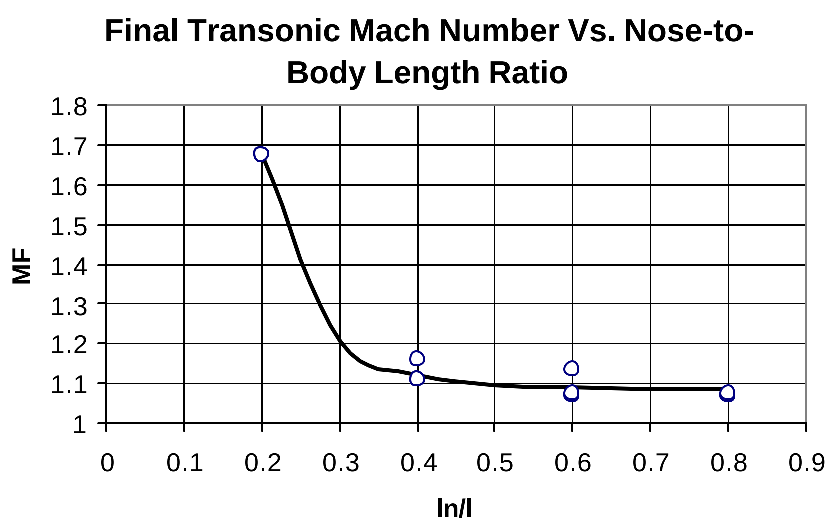


Figure 9 – Plot of Final Transonic Mach Number versus Nose-to-Body Length Ratio

The equation for ΔC_{DMAX} can be used to determine the best combination of nose-to-body length ratio (ln/l) and body length-to-diameter ratio (l/d) that would minimize wave drag rise for an equivalent body.

Let us consider using the equation to plot curves of wave-drag coefficient ($C_{Dw} = \Delta C_{DMAX}$) versus l/d; each curve at constant ln/l. Figure 10 illustrates such a family of curves. These curves suggest that regardless of ln/l, C_{Dw} will decrease with l/d. This suggests that for minimum wave drag rise one might consider a slender rocket body (nose cone + body tube). Recall that the Sears-Haack Body represents a minimum drag configuration at Mach =1.0 for ln/l =0.5. A curve describing the wave drag for the Sears-Haack Body is also plotted in Figure 10. This curve correlates well to those predicted using the equation for ΔC_{DMAX} when ln/l is in the vicinity of 0.4 to 0.5. For reference, an equation to predict wave drag coefficient for a Sears-Haack Body is given as follows:

$$C_{DW} = 1.125\left(\frac{pd}{l}\right)^2,$$

Where,

$d =$ Maximum diameter of Sears-Haack Body

$l =$ Total length of Sears-Haack Body

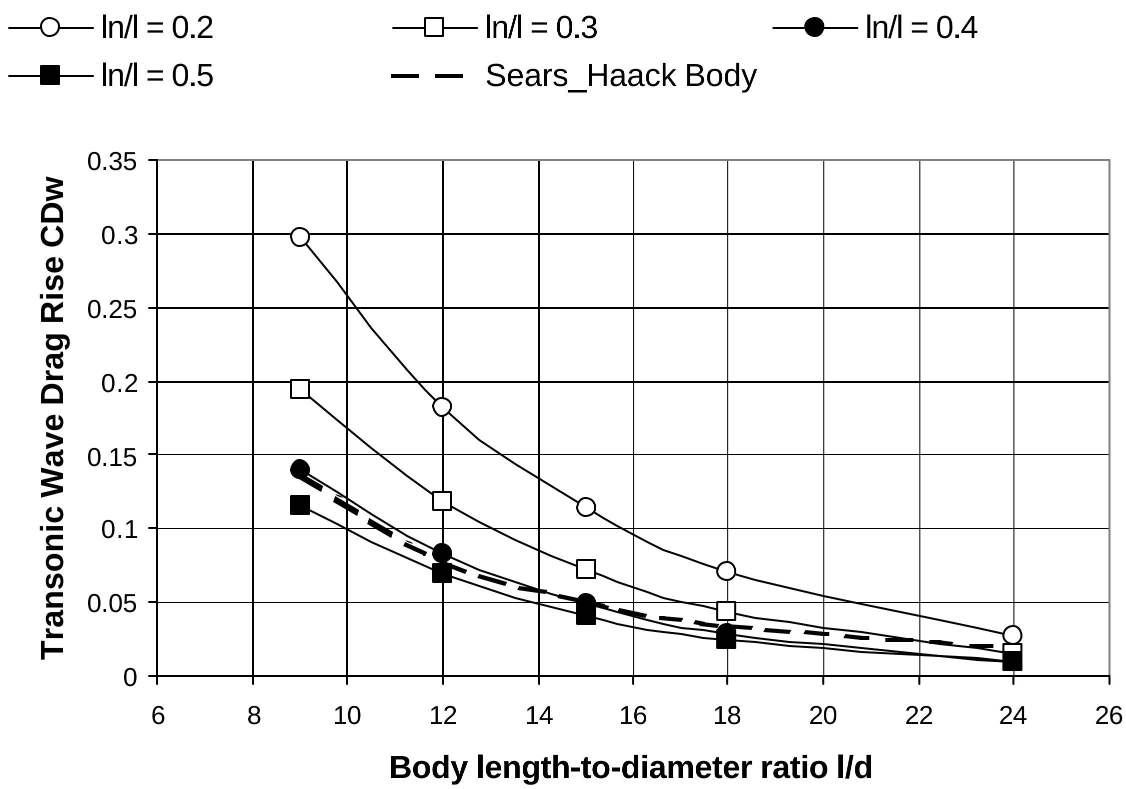


Figure 10 – Transonic drag rise versus body length-to-diameter ratio.

Let us consider plotting curves of C_{DW} versus ln/l; each curve at constant l/d. Figure 11 illustrates such a family of curves. These curves suggest that there is a minimum in wave drag as ln/l approaches 0.5. This may or may not be a practical design feature. Nevertheless, the trend indicates that by increasing ln/l (up to a limit of 0.5), the wave drag decreases.

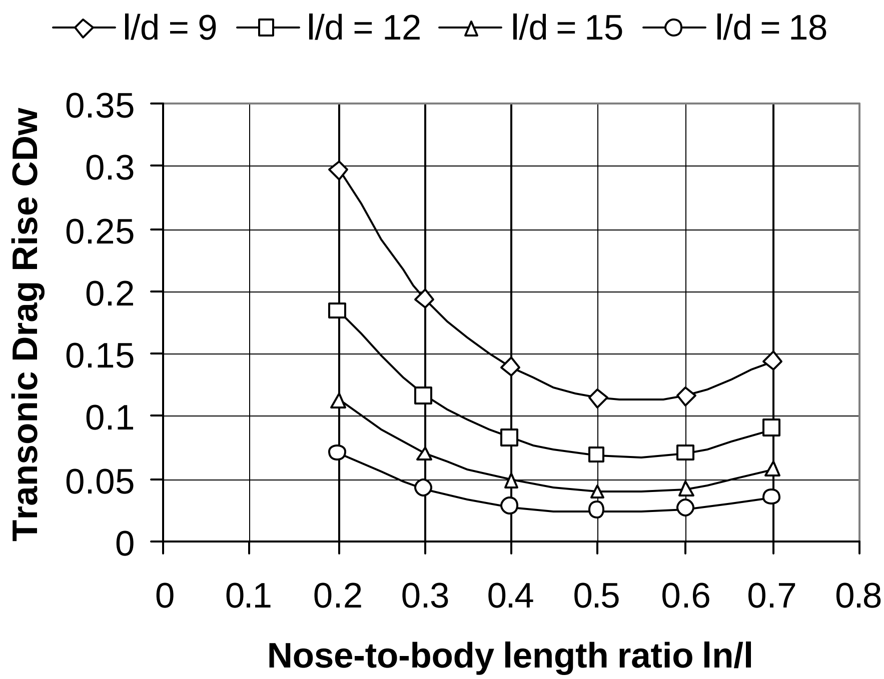


Figure 11 – Transonic Drag Rise Coefficient versus Nose-to-Body Length Ratio.

As a conclusion to the subject of designing for transonic flight, we can summarize a design approach to minimize the transonic wave drag rise. In short, the following can be considered:

1. The fin thickness-to-chord (t/c) ratio should be kept at or below 0.06. For structural reasons, I suggest a t/c no less than 0.04.
2. Avoid blunt leading and trailing edges. As a minimum requirement, the fin cross-section should have rounded leading and trailing edges. When designing for high speed, I prefer a symmetrical aerodynamic or a double wedge shape with the point of maximum thickness between 40% to 60% of the chord. The exact location will depend upon aerodynamic and structural considerations.
3. Use Area-Ruling. The equivalent body should possess a smooth and continuous cross-sectional area distribution along its longitudinal axis.
4. The larger the length-to-diameter ratio of the rocket's body (nose cone + body tube), the lower the transonic wave drag rise. Of course, there are practical limits.
5. Target a nose-to-body length ratio (ln/l) of 0.3 to 0.5. Recall that minimum transonic wave drag is associated with a ln/l of 0.5. Again, there may be practical limits to consider.

REFERENCES

1. Whitcomb, Richard T.: “A Study of the Zero-Lift Drag-Rise Characteristics of Wing-Body Combinations Near the Speed of Sound”, NACA RM L52H08, Langley Aeronautical Laboratory, Langley Field, Va., 3 September 1952.

2. Nelson, Robert L. and Welsh, Clement J.: “Some Examples of the Applications of the Transonic and Supersonic Area Rules to the Prediction of Wave Drag”, NACA RM L56D11, Langley Aeronautical Laboratory, Langley Field, Va., 20 March 1957.