Stability

Chapter 5

The primary purpose for having a statically and dynamically stable rocket is safety. A safe rocket flies straight and true, and in the intended direction of flight. Seldom do amateur rocket engineers equip their ships with active stability and control systems. Therefore, it is important that the rocket possess inherent stability. Generally, a rocket will possess its least amount of stability at liftoff weight, when the rocket's center-of-gravity is positioned most aft (toward the "business" end). A priority is that the rocket be designed with a healthy static stability margin (to be defined) at liftoff weight.

A second consideration for having a stable rocket is drag. Intuitively, one would think that larger fins would produce a more stable rocket; and the larger the fins, the greater the surface area, resulting in higher drag. This is not necessarily the case. If a rocket was marginally stable due to small fins and tended to fishtail during its flight, the drag developed during this wobbling motion can easily offset the amount of drag of a more stable rocket having slightly larger fins. So, there appears that there may exist an optimum fin configuration for each rocket design. It should be noted that the drag equations of Chapter 1 do not account for the additional drag induced by the non-optimum flight of a rocket continuously correcting its flight path. Therefore, it is not unexpected that the predicted altitude using the drag equations presented in Chapter 1 will be higher than in actual flight.

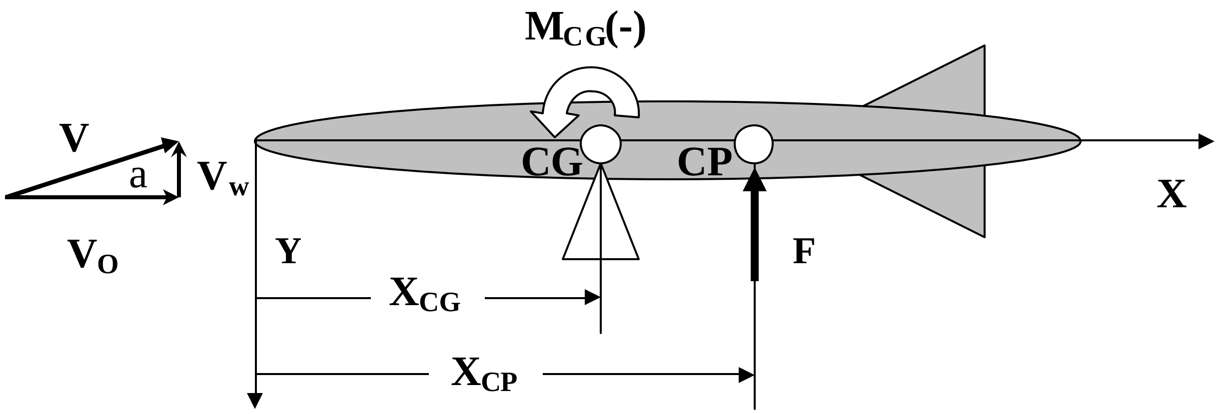
A rocket is not considered stable unless it meets both static and dynamic stability criteria. Although a rocket may be statically stable, it may not be dynamically stable. A statically stable rocket is a prerequisite for a dynamically stable rocket. This will become clear with the following discussions.

5.1 Static Stability –

A statically stable rocket will tend to return to its initial flight path after being perturbed from that flight path. The cause of this perturbation is usually a side force generated from a gust of wind. An unstable rocket will rotate about its Center-of-Gravity (CG). For this to occur, the rocket's lateral Center-of-Pressure (CP) must be located either forward or aft of the CG. The rocket's CP is the location of total applied side force with zero moment. However, a moment is generated at the rocket's CG equal to the product of the distance between the CP and the CG and the total side force. If the moment tends to restore the rocket's nose to its unperturbed state, it is a restoring moment and the rocket is statically stable. A statically stable rocket will have the CP located aft of the CG. If the moment tends to drive the rocket's nose away from its unperturbed state, it is a diverging moment and the rocket is statically and dynamically unstable. For this last case, the CP will be forward of the CG. For a "neutrally stable" rocket, the CP lies directly on the CG and the rocket's theoretical motion would be a pure lateral displacement.

Aerodynamicists define a restoring moment with a negative sign '-', and a diverging moment with a positive sign '+'. Aerodynamicists tend to be more concerned with rates. The convention is, a statically stable rocket will have a negative value for the rate of change in moment with change in angular displacement (angle-of-attack). A statically unstable rocket will have a positive value for the rate of change in moment with change in angle of attack. Designing to the criteria for a negative value of the rate of change in moment with change in angle-of-attack is a conservative approach compared to designing to ensure a restoring moment over a range of angle-of-attack.

To illustrate the discussion of static stability and instability, it is helpful to place a fulcrum at the rocket's CG. The following figure is an attempt to illustrate the concepts just described.



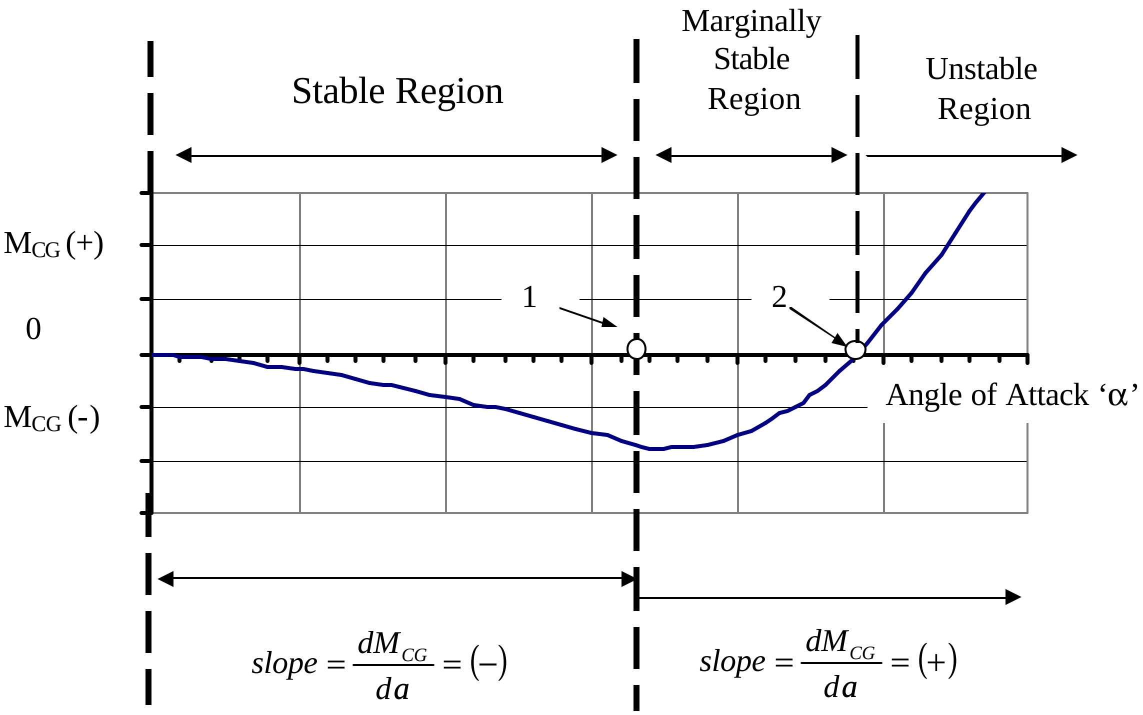
V_O = Velocity along flight path

V_W = Wind gust component acting normal to flight path

a = Angle of attack

$M_{CG} = F(X_{CP} - X_{CG})$ = Aerodynamic moment

F = Aerodynamic side force



The simplest rocket configuration typically has a body that is symmetric about its longitudinal axis, and the fins are aligned with zero geometric angle of incidence with this axis. If the longitudinal axis is co-linear with the flight path and there is no crosswind component (i.e.; gust induced), then the rocket's total velocity vector is aligned with its longitudinal axis, defining a zero angle-of-attack. In reference to the above plot of M_{CG} versus a, the situation just described corresponds to the origin point 0. Now, let us suppose that a gust creates a crosswind component that induces an angle-of-attack a. This rotates the rocket's total velocity vector relative to its longitudinal axis about the CG, the amount equal to a. As a increases, the CP location moves forward toward CG location. If a is less than that of Point 1 indicated on the plot, then the moment M_{CG} is negative, the slope of the curve is negative, and the rocket returns to its original position. Hence, the rocket is stable. There is a small region of angle of attack 'a', between Points 1 and 2 of the plot, where there exists a restoring moment (M_{CG} is negative) but the slope of the M_{CG} versus a curve is positive. In classical aerodynamics, a positive slope is considered an unstable state. However, as long as a remains less than that of Point 2, there will exist a restoring moment that will tend to restore the rocket to a zero angle-of-attack. I refer to the region between Points 1 and 2 as marginally stable. Point 2 corresponds to an angle-off-attack 'a' where the rocket's center-of-pressure (CP) location has moved to the rocket's center-of-gravity (CG) location, resulting in a state of neutral stability. Beyond Point 2 the aerodynamic moment (M_{CG}) becomes positive and the rocket will continue to diverge from its original state

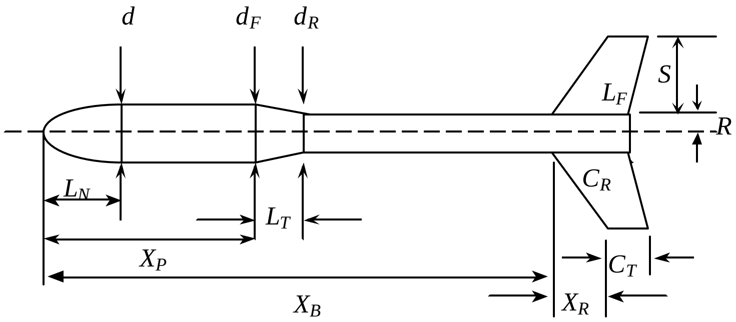
Obviously, it should be a design requirement to have the rocket's CP aft of its CG to guarantee an inherently statically stable rocket. A recommended distance of the CP aft of the CG is equal to one rocket body diameter `d` at liftoff weight. The CG location X_{CG} is easily found by finding the rocket's balance point when configured for flight at liftoff weight. The CP location X_{CP} can be estimated using the Barrowman Equations (reference 1). The stability margin SM is defined as the distance between the CP and CG locations normalized with respect to the body diameter d. Symbolically, this is described as:

$$SM = \frac{(X_{CP} - X_{CG})}{d} = Static\ Stability\ Margin$$

If X_{CP} is greater than X_{CG}, then SM is positive and the rocket is statically stable. If X_{CP} is less than X_{CG}, then SM is negative and the rocket is statically unstable. If X_{CP} is equal to X_{CG}, then SM is zero and the rocket is neutrally stable. As previously mentioned, it is suggested that SM have a value near unity.

James and Judith Barrowman published a paper entitled, "The Theoretical Prediction of the Center of Pressure – NARAM-8 R&D Project", that describes a set of equation religiously used for the prediction of rocket X_{CP}. The report was reprinted by Apogee Components. It can be downloaded from their web site: www.ApogeeRockets.com. Their equations are reproduced below:

Geometry -



Where,

L_N = *length of nose*
d_F = *initial diameter of transition*
d_R = *aft diameter of transition*
L_T = *distance from d_F to d_R*
X_P = *distance from nose to transition*
d = *aft diameter of nose*
C_R = *fin root chord*
C_T = *fin tip chord*
S = *half-span of fin*
L_F = *length of fin mid-chord line*
R = *body radius at aft end*
X_R = *Longitudinal distance between fin root leading edge and fin tip leading edge*
X_B = *Distance from nose to fin leading edge at root chord*

Normal Force (side force) Coefficients -

$(C_N)_N$ = *normal force coefficient for nose*
= *2.0*
$(C_N)_T$ = *normal force coefficient for transition*

$$= 2\left[\left(\frac{d_R}{d}\right)^2 - \left(\frac{d_F}{d}\right)^2\right]$$

$(C_N)_F$ = *Normal force coefficient for fin group*

$$= \left[1+\frac{R}{S+R}\right]\left[\frac{12\left(\frac{S}{d}\right)^2}{1+\sqrt{1+\left(\frac{2L_F}{C_R+C_T}\right)^2}}\right] \text{ for 3 fins}$$

$$= \left[1+\frac{R}{S+R}\right]\left[\frac{16\left(\frac{S}{d}\right)^2}{1+\sqrt{1+\left(\frac{2L_F}{C_R+C_T}\right)^2}}\right] \text{ for 4 fins}$$

C_N = *total normal force coefficient*
= $(C_N)_N + (C_N)_T + (C_N)_F + \ldots$

Note: For multiple groups of fins and/or conical transitions, a normal force coefficient is calculated for each group. The rocket's total normal force coefficient is the summation of each group coefficient plus that of the nose section.

Center of Pressure Locations -

$X_N =$ *center of pressure location for nose section*

$= 0.666L_N$, *for a cone shaped nose*

$= 0.466L_N$, *for an ogive shaped nose*

$X_T =$ *center of pressure location for transitions*

$$= X_P + \frac{L_T}{3}\left[1 + \frac{1 - \dfrac{d_F}{d_R}}{1 - \left(\dfrac{d_F}{d_R}\right)^2}\right]$$

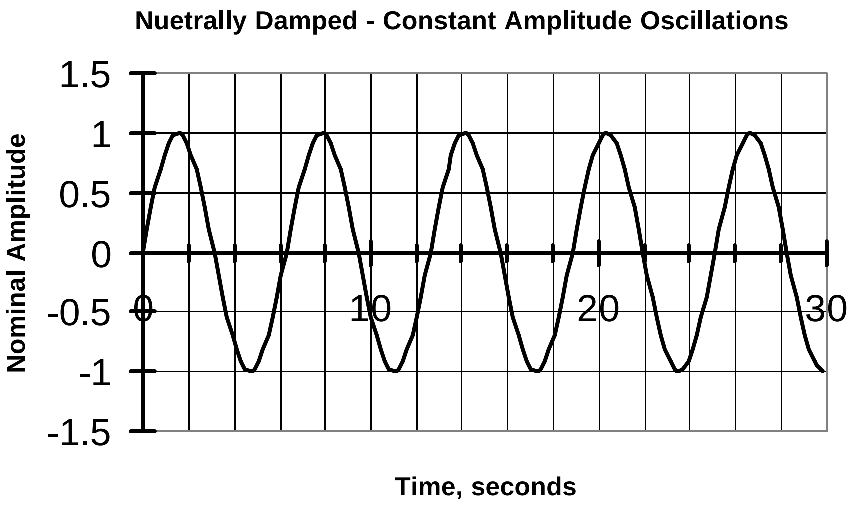
$X_F =$ *center of pressure location for fin groups*

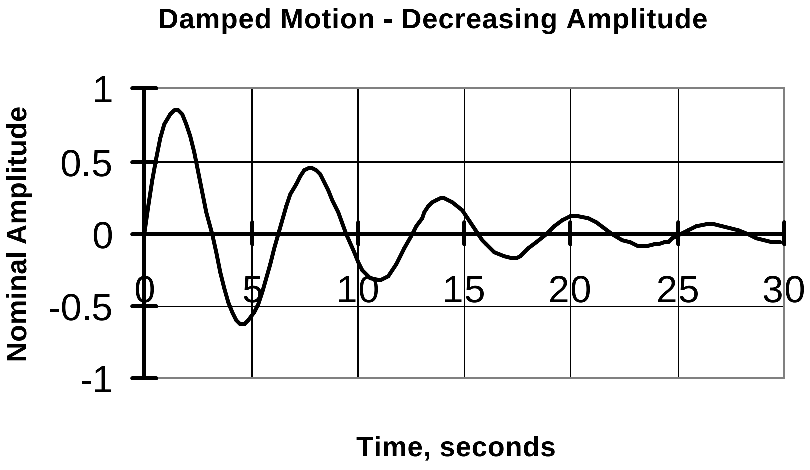
$$= X_B + \frac{X_R}{3}\frac{(C_R + 2C_T)}{(C_R + C_T)} + \frac{1}{6}\left[(C_R + C_T) - \frac{(C_R C_T)}{(C_R + C_T)}\right]$$

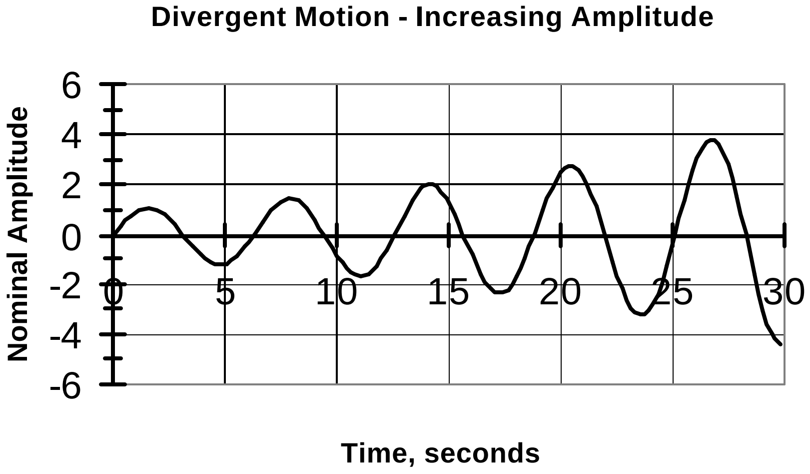
$$\longrightarrow \quad X_{CP} = \frac{(C_N)_N X_N + \ldots + (C_N)_T X_T + \ldots + (C_N)_F X_F + \ldots}{C_N}$$

5.2 Dynamic Stability –

A prerequisite to dynamic stability is static stability; that is, the rocket must exhibit an initial tendency to return to its flight path after being perturbed. Once static stability is satisfied, the rocket's gross motion may be characterized by an oscillation about its flight path. If the amplitude of this oscillation decreases with time, the motion is damped. If the amplitude of the oscillation increases with time, the motion is divergent. If the amplitude of the oscillation remains constant with time, the motion is neutrally damped; neither damped nor divergent. Obviously, it is desirable for the rocket to have damped motion. The following plots of a nominal variable describing a given motion (mode) illustrate the basic concepts of neutrally damped, damped, and divergent motion.







Flight vehicles will experience several characteristic modes of motion as open loop responses. That is, when the vehicle is perturbed from steady state it usually responds in a characteristic mode of motion unassisted by active control. Some of these motions can be rather complex and involve the interaction of several degrees-of-freedom. For rigid body motion, these degrees of freedom are the three orthogonal linear velocity components (axial, vertical and lateral) and the three angular velocities of roll, pitch and yaw rate, and the three angular displacements of roll, pitch and yaw. One such coupled motion often associated with motion sickness is the Dutch Roll, which involves the interaction of roll and yaw degrees of freedom.

A flight vehicle's open loop response can be characterized mathematically with the help of Complex Numbers (numbers having real and imaginary parts). The equations of motion are derived and then "linearized" to characterize the vehicle's response in terms of normalized perturbation variables associated with the above described degrees-of-freedom. For a prescribed disturbance, the equations are solved using linear algebraic techniques. The solution usually involves the description of the overall motion by one complex number known as an eigenvalue. This eigenvalue can be further decomposed into an array consisting of the perturbed degrees-of-freedom. This array is known as an eigenvector. This set of complex numbers can provide an engineer with detailed information about the vehicle's response.

The type of linear analysis just described is usually performed for a flight vehicle that has been perturbed about a steady state condition. Usually this is a constant velocity flight profile. The analysis to do this is beyond the scope of this text. A rocket's flight profile is not at constant velocity and weight.

Velocity and weight are continuously changing. With a change in weight, there is a change in CG-location and mass moment-of-inertia; both important to the rocket's stability. This would further complicate the prediction of the rocket's dynamic response.

Mr. Edward V. LaBudde (NAR #73451) submitted a report to NARAM in August of 1999, describing an R&D project for development of "A Design Procedure for Maximizing Altitude Performance". Mr. LaBudde describes a method of reducing the complex 6 degree-of-freedom dynamic stability problem down to 2 degrees-of-freedom. He first assumes a zero roll rate system, thus uncoupling the longitudinal and lateral equations of motion. This reduces the problem to 3 degrees-of-freedom. Second, small angles were assumed (typically less than 15^{o}), that reduces a 3 degree-of-freedom problem to a 2-degree of freedom problem. It is an objective of the paper to estimate the loss in altitude due to the non-optimum trajectory of the rocket when encountering crosswind conditions.

Short of sharpening the pencil, opening a textbook and firing up a computer, what can we do to design our rocket with dynamic stability? Recall that damping is important to dynamic stability. Damping is usually improved by an increase in fin surface area and a healthy distance between the CG-location and CP-location; the CP-location being aft of the CG-location. Dynamic stability will be enhanced by designing to a reasonably large static margin SM. Again, it is suggested that a SM approaching 1.0 be a design goal.

REFERENCES

1. Barrowman, James and Judith; "The Theoretical Prediction of the Center of Pressure – NARAM-8 R&D Project"; www.ApogeeRockets.com

2. LaBudde, Edward V.; "A Design Procedure for Maximizing Altitude Performance", Research and Development Project submitted at NARAM, August 1999.