Parachute Recovery Analysis

Chapter 4

We will discuss the derivation of an equation that predicts the constant sink speed of a rocket under the recovery of one or more parachutes. After plowing through several pages of what might be considered boring hocus-pocus, you will be rewarded with example calculations of a real life rocket recovery problem. First you must pay the price of enlightenment, or whatever, so here goes …

As with the discussion on rocket drag coefficient, we will start with the equation for dynamic pressure. In equation form this is:

$$q = \frac{1}{2} r V^2$$

Where,

r = *Air density, lb.-sec*^{2} */ ft*^{4}

V = *Flow velocity, ft/sec*

Recall the example of putting your hand or a flat plate out the window of your car such that the flat surface is position normal to the airflow. The total force required to keep it in position when impinged by the oncoming airflow will be approximately equal to that defined above. Realistically, the actual force is dependent on the shape of the object, and the 3-dimensional flow characteristics of the fluid (i.e.; flow relief, turbulent and/or laminar flow, etc.). Often these influences are summed up in a single coefficient known as an aerodynamic coefficient. In our particular case, we are interested in a drag force, that is $F = D$, and likewise a drag coefficient C_D.

Depending on equation formulation and reference area, C_D can take on different values. This paper relies heavily on the C_D data as published by Top Fight for their parachutes. The C_D for Top Flight is based on a circular reference area given by the flat chute diameter. However, the equations that are derived here attempt to account for the option of a spill hole. Therefore, initially we will use a constant of proportionality designated by K, rather than C_D. Later K will be converted to C_D. In general, the equation for drag 'D' is given by:

$$D = qKA$$

Where,

q = *Dynamic pressure, lb. / ft*^{2}

K = *Constant of proportionality*

A = *Reference area, ft*^{2}

In this formulation the constant of proportionality 'K' is based on a reference area 'A' given by the chute deployed radius 'R_o'. The reference area is simply the area of the circle described by this radius. The drag equation becomes:

$$D = qKA = qKpR_o^2$$

Where,

p = *Constant Pi » 3.14159*

R_o = *Chute deployed radius, ft*

To account for the reduction in drag due to a spill hole, a quantity of drag is subtracted from the above equation that is referenced to the deployed radius of the hole, given by r_o.

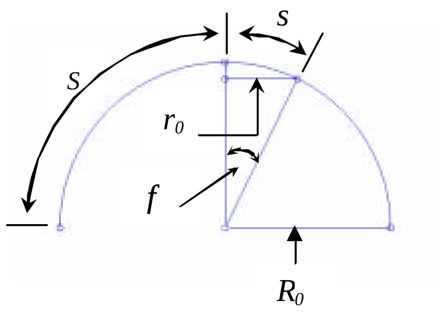
$$D\,(With\ Hole) = qKpR_o^2 - qKpr_o^2$$

$$Where,$$
$$r_o = Deployed\ radius\ of\ spill\ hole,\ ft$$

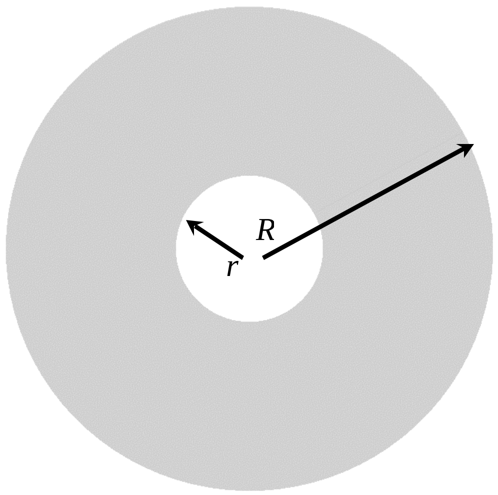
Now, several bold assumptions are made. The first is that the dynamic pressure 'q' does not decrease as the flow approaches the spill hole. Generally one would expect a decrease in q due to energy losses caused by mechanisms such as friction drag. Also a buildup of boundary layer might effectively reduce the value of r_o. On the other hand, another assumption is that K at the spill hole is unchanged from that of a chute without a spill hole. Although, one might expect that K would increase due to turbulent flow separation and flow entrapment as the air passes around the edge of the spill hole. Whether right or wrong, and without any substantiation, these assumptions are being employed simply for the ease of equation definition. The chute drag equation becomes:

$$D\,(\text{With hole}) = qKp\,(R_o^2 - r_o^2)$$

The above equation is not very useful when expressed in terms of the deployed radii. Typically, the deployed radii are not known. It is better to express the drag equation in terms of the chute flat radii, because these can be easily measured. The deployed radii are converted to the flat chute radii as follows:



Chute Flat Radii



From the geometry:

$$R = S = \frac{R_o p}{2} \longrightarrow R_o = \frac{2R}{p}$$

$$\sin(f) = \frac{r_o}{R} \longrightarrow f = \sin^{-1}\left(\frac{r_o}{R_o}\right)$$

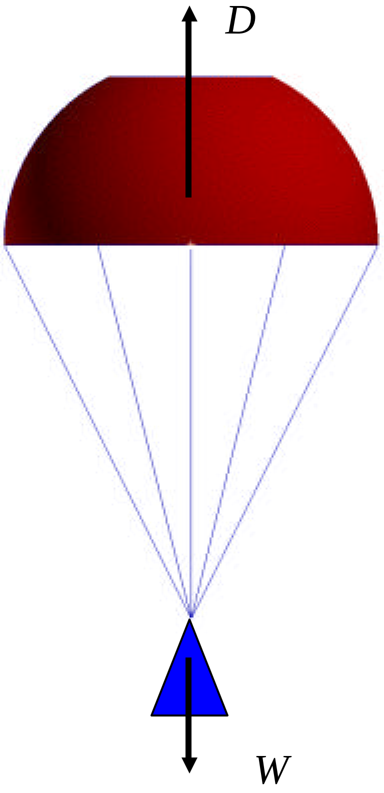
$$r = s = R_o f \longrightarrow r = R_o \sin^{-1}\left(\frac{r_o}{R}\right)$$

$$\therefore \sin\left(\frac{r}{R_0}\right) = \frac{r_0}{R_0} \longrightarrow r_0 = R_0 \sin\left(\frac{r}{R_0}\right) = \frac{2R}{p}\sin\left(\frac{rp}{2R}\right)$$

After the above relations for r_0 and R_0 are substituted into the drag equation, and some algebraic hocus pocus is performed, the equation for drag with a spill hole is:

$$D = \frac{4qKR^2}{p}\cos^2\left(\frac{rp}{2R}\right)$$

Before moving onto the development of the equation for prediction of constant sink speed under multiple parachutes, this would be a good time to define the constant '*K*' in terms of the drag coefficient 'C_D'. First, let us consider the use of only one parachute with no spill hole. At constant sink speed, acceleration must be zero and the summation of forces must equal zero. Per the following diagram, the only forces acting on the system are weight '*W*' and drag '*D*'.



$$\sum F = 0 = W - D$$

$$\longrightarrow \quad W = D$$

The drag is given by the equation,

$$D = qC_D A = qC_D pR^2 .$$

Now, the drag for a parachute without spill hole is also given by the drag equation for a chute with spill hole, but with the radius set to zero. That is,

$$D = \frac{4qKR^2}{p}\cos^2\left(\frac{\cancelto{0}{rp}}{2R}\right) = \frac{4qKR^2}{p}\cos^2(0) = \frac{4qKR^2}{p}$$

Equating the two drag equations gives,

$$\frac{4qKR^2}{p} = qC_D pR^2 \quad \longrightarrow \quad K = \frac{p^2 C_D}{4}.$$

The relation for K can be substituted back into the drag equation that accounts for spill hole radius, to describe drag in terms of C_D.

$$\therefore D = qpC_D R^2 \cos^2\left(\frac{rp}{2R}\right)$$

The above equation describes the drag for one chute only. In the event that there are multiple chutes in the recovery of the rocket, it is assumed that their individual drag contributions are added. This assumes that there are no losses due to interference of the chutes. As in the previous diagram, for constant sink speed the sum of the drag contributions must equal the total weight. In equation form, we say:

$$W = D_1 + D_2 + D_3 + \ldots = \sum_i^N D_i$$

Where,

$i = 1 \ldots N$, *Chute number*

$N =$ *Total number of chutes*

To solve for the sink speed, we first substitute the relation for drag into the above equation and collect like terms.

$$W = \sum_i^N D_i = \sum_i^N \left[qpC_D R^2 \cos^2\left(\frac{rp}{2R}\right)\right]_i = qp\sum_i^N \left[C_D R^2 \cos^2\left(\frac{rp}{2R}\right)\right]_i$$

Recall the relation for the dynamic pressure 'q',

$$q = \frac{1}{2} rV^2.$$

We substitute the relation for q into the bodacious equation above, and solve for V. We now have the equation to predict sink speed! It is presented below for your enjoyment.

$$V = \left[\frac{2W}{rp\sum_i^N \left[C_D R^2 \cos^2\left(\frac{rp}{2R}\right)\right]_i}\right]^{1/2} \text{, In units of ft/s}$$

Where,

$r =$ *Air density, lb.-sec*^{2} */ ft*^{4}

$r = .002378$ *at sea level standard conditions*

p = *Constant Pi » 3.14159*
W = *Rocket burnout weight, lbs.*
R = *Flat chute radius of each chute, ft*
r = *Flat radius of spill hole of each chute, ft*
C_D = *Drag coefficient of each chute*
i = *Chute number; 1, 2 … N*
N = *Total number of chutes*

Before moving onto example calculations, it would be nice to know some typical values of C_D for various chute configurations. Below is a table of C_D values for various chute types.

Type of Parachute	Typical Range of C_D
Flat Circular	0.75 – 0.80
Hemispherical	0.62 – 0.77
X-Form	0.60 – 0.85
Conical	0.75 – 0.90
Bi-Conical	0.75 – 0.92
Tri-Polyconical	0.80 – 0.96

It is time to do some calculations! We will start simple. The first problem will be for a single chute recovery using a hemispherical chute. In this example, we will study the effects of varying the spill hole radius. For a single chute, the equation reduces to:

$$V = \left[\frac{2W}{r p C_D R^2 \cos^2\left(\dfrac{rp}{2R}\right)} \right]^{1/2}$$

The inputs for this example problem are:

$r = .002378$ lb.-sec^2 / ft^4
$p = 3.14159$
$W = 127$ lbs.
$R = 14$ ft
$r = 0, 2, 4, 6, 8, 10$
$C_D = 0.75$

$$\longrightarrow V = \left[\frac{(2)(127)}{(.002378)(p)(0.75)(14)^2 \cos^2\left(\dfrac{rp}{2R}\right)} \right]^{1/2} = 15.208\left[\cos\left(\frac{rp}{2R}\right)\right]^{-1}$$

Finally, after substituting in the various values of spill hole radii ‘*r*’, the predicted sink speeds are:

r	$\left[\cos^2\left(\frac{rp}{2R}\right)\right]^{-1}$	V
0 ft	1	15.21 ft/s
2 ft	1.052	16.00 ft/s
4 ft	1.110	16.88 ft/s
6 ft	1.279	19.45 ft/s
8 ft	1.604	24.39 ft/s
10 ft	2.305	35.06 ft/s

Another form of the sink speed equation for a single chute that eliminates the need to do a trigonometric calculation is:

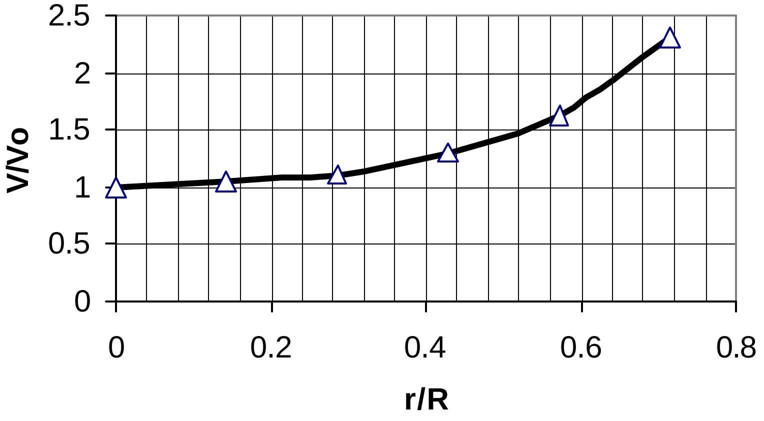
$$V = \left[\frac{2W}{rpC_D R^2}\right]^{1/2}\left(\frac{V}{Vo}\right), \text{ in units of ft/s}$$

Where,

$\left(\frac{V}{Vo}\right)$ = *Ratio of speed with hole to speed without hole.*

Its value can be determined from the following graph for given r/R ratios.

Sink Speed Ratio Vs. Spill Hole-to-Chute Ratio



It would be fun to run a simple experiment to validate or invalidate the equation. This experiment could be as simple as dropping chutes with weights from an adequate known height to establish a constant sink speed for most of the descent. The chutes could have various *r/R* ratios. The average sink speed could be calculated by dividing the drop height by the time of descent. Each configuration should be tested several times in order to establish a reliable average of sink speed for that configuration. The baseline configuration would be a chute without a spill hole, that is *r* = 0. The baseline sink speed is denoted by *Vo*. Then for each chute configuration of *r/R*, the *V/Vo* ratio is its average sink speed divided by the baseline’s sink speed. It would be interesting to compare the experimental results to the theoretical results.

Now, let us atempt a multi-chute recovery calculation. We will use the same 127-pound rocket, but this time it will have a 4-foot drogue chute of X-plan form and three 10-foot hemispherical chutes with no spill holes. The drogue chute will be deployed at 7300 feet altitude above sea level, and the three main chutes will be deployed at 1000 feet. We are interested in determining the average sink speed and time between 7300 feet and 1000 feet altitude, and the remaining descent below 1000 feet. An average wind speed of 7.5 ft/s is expected that day, so it is of interest to determine how far the rocket might drift during descent. First, let us determine the sink speed under the deployment of a single X-plan form chute. The chute does not have a spill hole. Therefore, the sink speed equation reduces to:

$$V = \left[\frac{2W}{r p C_D R^2}\right]^{1/2}$$

The inputs for calculation of the sink speed using the single X- plan form chute are:

$W = 127\ lbs.$
$r = .002136\ lb\text{-}sec^2 / ft^4$ *for an average altitude of 4150 ft*
$C_D = 0.6$
$R = 2\ ft$

$$\longrightarrow \quad V = \left[\frac{2(127)}{(.002136)p(0.6)(2)^2}\right]^{1/2} = 126\, ft/s$$

The descent time and horizontal distance traveled are given by:

$$t(drogue) = \left[\frac{(7300-1000)\, ft}{126\, ft/s}\right] = 50s$$

$$d(drogue) = (7.5\, ft/s)(50s) = 375\, ft$$

We better slow this monster down in the last 1000 ft of descent so we can fly another day. If all goes as planned, three 10-ft chutes should deploy along with the already abused drogue chute. Again, without center holes, the sink speed equation reduces to:

$$V = \left[\frac{2W}{p\, r \sum_{1}^{4} (C_D R^2)_i}\right]^{1/2} = \left[\frac{2W}{p\, r\left((C_D R^2)_1 + (C_D R^2)_2 + (C_D R^2)_3 + (C_D R^2)_4\right)}\right]^{1/2}$$

For our problem, all three main chutes are identical. If we replace the subscripts 1 through 3 by ‘m’ , denoting main chute, and the subscript 4 by ‘d’, denoting the drogue chute, the equation simplifies to:

$$\longrightarrow \quad V = \left[\frac{2W}{p\, r\left((3C_D R^2)_m + (C_D R^2)_d\right)}\right]^{1/2}$$

Now, the inputs for our problem are:

$W = 127\ lbs.$
$r = .002356\ lb\text{-}sec^2 / ft^4$ *for an average altitude of 500 ft*
$C_D = 0.6$, *for the drogue chute*
$C_D = 0.75$, *for the main chutes*
$R = 2\ ft$, *for the drogue chute*
$R = 5\ ft$, *for the main chute*

$$\longrightarrow \quad V = \left[\frac{2(127)}{p\,(.002356)\left(3(0.75)(5)^2 + (0.6)(2)^2\right)}\right]^{1/2} = 24.19\,ft/s$$

Again, we calculate the descent time and horizontal distance traveled as follows:

$$t(main) = \left[\frac{1000\,ft}{24.19\,ft/s}\right] = 41.3s$$

$$d(main) = (7.5\,ft/s)(41.3s) = 310\,ft$$

$$\longrightarrow \quad t(total) = t(drogue) + t(main) = 50 + 41.3 = 91.3s$$

$$\longrightarrow \quad d(total) = d(drogue) + d(main) = 375 + 310 = 685\,ft$$

Let us pray that the main chutes open as planned!